Understanding Flywheel Energy Storage: Does High-Speed Really Imply a Better Design?

White Paper 112

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Objective

This paper will review how energy is stored in a flywheel using the simple concept of a massive ball attached to a limited strength string. This concept will also be used to better understand the relationship between flywheel mass and strength properties. The paper will discuss how material strength influences the performance attributes of flywheels, examining two types of materials – steel and graphite fiber reinforced epoxy (GFRE).

A manufacturer of high-speed flywheel energy-storage systems for uninterruptible power supply (UPS) applications states the following:

"Kinetic energy is roughly equal to mass times velocity squared. So doubling mass doubles energy storage, but doubling the rotational speed quadruples energy storage."

The implication of this statement is that high speed flywheels are superior to low speed designs. The truth is that this statement misses several important facts about the physical limitations faced by flywheel designers and is thus not sufficient for even the most basic comparison of flywheel designs.

The above statement is based on the equations for energy storage of a body of *mass* (m) which is moving in a straight line with a *velocity* (v).

$$E = \frac{1}{2} \cdot m \cdot v^2 \tag{1}$$

However, instead of operating as a mass moving in a straight line, commercial flywheels spin around a central axis. Instead of using linear velocity as mentioned above, one should analyze flywheel performance as a rotating body. The term *angular velocity* (ω) is used to define the rotational speed of the flywheel and is represented by the Greek letter omega. A discussion of this term is given in Appendix A.

One-Dimensional Flywheel Design

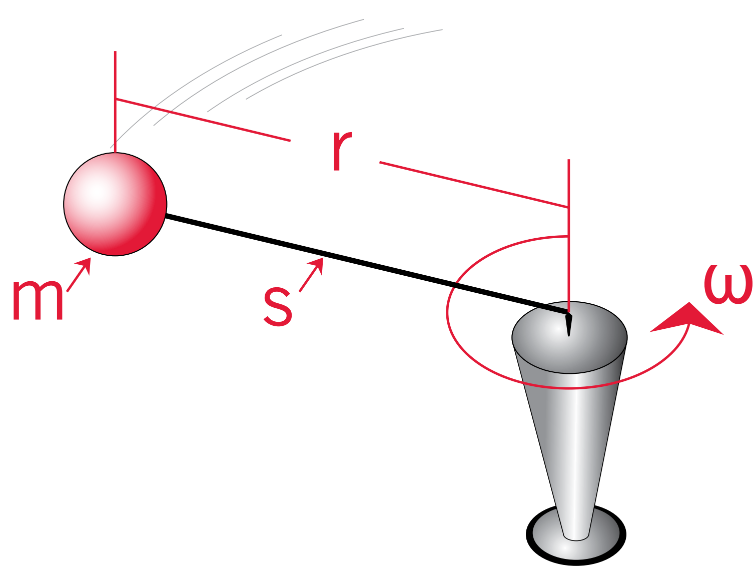

Figure 1: One-dimensional flywheel geometry.

In order to simplify the discussion of flywheel design, consider the simplest one-dimensional flywheel – that of a ball having a lumped mass (m) attached to a central point by a string having a length (r), and strength (S) as shown in Figure 1. If one spins this “flywheel” about the central point, the mass will have an angular velocity (ω). The linear velocity (v) of the mass will be in a direction perpendicular to the string and will be equal to the string length (radius of the circular path) times the angular velocity giving $v = r\omega$. If the equation for velocity is substituted into the energy expression above, one sees that the new expression for the stored energy of a one-dimensional (lumped mass) flywheel is:

$$E = \frac{1}{2} \bullet m \bullet (r \bullet \omega)^2 = \frac{1}{2} \bullet m \bullet r^2 \bullet \omega^2 \qquad (2)$$

At this point, it still appears that the kinetic energy statement above is valid even though the terms from linear motion have been changed to those more applicable for rotary motion. But now, consider what is happening to the string that maintains the rotary motion.

Most individuals are familiar with the concept of centrifugal force - spin a weight on the end of a string and it takes more force to hold onto the string the faster it is spun. Spin it fast enough and the applied force on the string exceeds the strength of the string and the string breaks. The simplified mass of the one-dimensional flywheel represents the energy storing potential while the string represents the strength of the flywheel material. The expression for the centrifugal force exerted by a mass spinning around a central point is:

$$F = m \bullet r \bullet \omega^2 \qquad (3)$$

Note that Equation 3, the force on the string, is a direct function of the radius of the mass whereas in Equation 2, the stored energy is a function of the radius squared. One is faced with another observation from the one-dimensional flywheel:

“Doubling the mass doubles the force, but doubling the speed quadruples the force.”

Thus, the original kinetic energy statement is true only if the strength of the material being used is capable of bearing the increase in applied forces. Obviously, if this were the case, the designer of the original flywheel has left performance capability on the table with a poorly optimized design. Flywheel designers always strive for designs that operate at the highest possible energy *within the safe performance constraints of the available materials*. But how do actual material properties affect the results?

Assuming one is comparing one-dimensional flywheels of similar geometry (a reasonable assumption since the influence of material properties is being compared, not shapes), the radius of the two designs would remain fixed and equally important, the cross sectional area of the string would remain fixed. If the cross sectional area (A) of the string is calculated, one can determine the stress applied to the string. The Greek letter sigma is used to represent stress as follows:

$$\sigma = F / A \qquad (4)$$

At this point, enough information exists to compare one-dimensional flywheels made from the two most common materials for high performance flywheels – steel and GFRE (graphite fiber reinforced epoxy).

A variety of steels have been used flywheels for energy storage applications. While some slight variation in density (weight per unit volume) for different steel alloys does exist, the value tends to be close to 0.28 to 0.29 pounds per cubic inch. For GFRE materials, the density is a composite of the graphite density and the epoxy density. For flywheels, the ratio of graphite fiber to epoxy is on the order of 60 to 70 percent giving a composite density of 0.05 to 0.06 pounds per cubic inch. If one did not to take advantage of the higher strength properties of a GFRE flywheel, there would be no advantage to using this material. In fact the energy storage would be lower by nearly a factor of six according to Equation 2 for a given geometry and speed.

In order to complete the example, material strength must be compared. Again, the strength of both steel and GFRE can vary, and considerably more so than the density. Plain carbon steels with no heat treatment can have strength values in the low tens of thousands of pounds per square inch (psi) compared to highly alloyed and/or heat treated materials with strength values of 500,000 psi or more. At these ultrahigh strength levels, the materials cannot be economically processed nor do they have properties amenable to safe incorporation into flywheels. Therefore, this discussion will assume that the limiting strength of this one-dimensional steel flywheel is 180,000 psi.

GFRE material strength varies considerably depending on the properties of the carbon fiber and on the volume fractions of fiber and epoxy compared to the total volume. Ultra-high strength fibers are available with strength values approaching one million psi. Fibers in this range cost approximately $135 per pound whereas fibers with strength closer to 800,000 psi are more economical at $50 per pound. When raw graphite fibers are mixed with epoxy to form a layered structure, the strength of the “laminate” is reduced roughly in proportion to the volume of fiber to the total volume of fiber and epoxy in the direction of loading. As indicated above, the volume fraction is typically between 60 and 70 percent giving a net strength of approximately 500,000-psi for the laminate. Tremendous room for tradeoff exists between GFRE laminate performance and cost that a designer must evaluate. Furthermore, one must carefully evaluate the benefits of increased performance offered by GFRE composites given the raw material cost of high quality steels is substantially lower at about $1 per pound.

To compare the performance of the one-dimensional flywheels based on the use of the two materials identified, assume the mass of the flywheel is concentrated in a sphere of one inch diameter, the string length is one foot (12-inches), and the string has a diameter of approximately 1/32 of an inch (0.032-inch) – about the diameter of a pin. Results of this comparison are given in Table 1.

Mass diameter	1 in
Mass volume	0.524 in3
Radius	12 in
String diameter	0.03 in
String x-section area	0.000707 in2

Material	Material Density (lb/in^3)	Material Strength (lb/in^2)	Material Cost ($/lb)	Weight (lb)	Breaking Load (lb)	Angular Velocity to Break (rad/sec)	Rotational Speed to Break (rev/min)	Stored Energy (watt-sec)	Flywheel Cost ($)
Steel	0.283	180,000	$1.00	0.148	127	166.3	1,588	86	$0.15
GFRE	0.058	500,000	$50.00	0.030	353	612.2	5,846	240	$1.52

TABLE 1: ONE-DIMENSIONAL FLYWHEEL PERFORMANCE COMPARISON.

For each material, the breaking strength (S) of the string is first calculated using Equation 4 by multiplying the material strength (σ) by the string cross-sectional area. Using Equation 3, angular velocity (ω) required for mass (m) spinning at radius (r) to generate the breaking load (F=S) is calculated. Angular velocity is converted into rotational speed using the more conventional units of revolutions per minute for reference. Next, using Equation 2, the stored energy (E) from the spinning mass (m) rotating at radius (r) with angular velocity (ω) is calculated. Finally, the cost of both flywheels using the cost per unit mass multiplied by the mass (m) is calculated.

The results show the stored energy of the GFRE flywheel is indeed higher than one manufactured from steel. Notice the ratio of the breaking limit angular velocities of the two materials is 612.2 divided by 166.3 or 3.68. Using the “speed squared principle” of the original kinetic energy statement as the basis for comparison suggests that the energy storage ratio should be equal to this amount squared or 13.55. However, the actual ratio of stored energy including the inherent properties and limitations of the materials in question is only 2.77.

A few additional points can be extracted from the results in Table 1. If one divides the stored energy by the weight, one obtains a *gravimetric energy density* expressed in units of energy storage per unit mass such as watt-seconds per pound or per kilogram. Usually, the term gravimetric is dropped and the result is simply referred to as the energy density*. The energy density of the steel flywheel in this example is 1,169 watt-seconds per pound and that of the GFRE flywheel is 15,967 watt-seconds per pound. This example illustrates one of the more attractive properties of using GFRE for flywheels compared to steel and that is the improved energy density.

One should now consider the cost of these flywheels on a per unit energy basis. For the steel flywheel, dividing the cost of $0.15 by 173 watt-seconds stored energy gives a specific cost of $0.867 per kW-second. Similarly, for the GFRE flywheel, dividing the cost of $1.52 by 479 watt-seconds stored energy gives a specific cost of $3.17 per kW-second; a factor of 3.66 times higher. When considering the use of flywheels for aerospace or ground vehicle applications, efforts to maximize the energy density (maximum energy per unit weight) can often justify the higher costs of these materials. However, for stationary ground-based applications, the value of energy density generally receives lower priority and the final decision on which material to use is better left to detailed economic comparison of the final system and other performance attributes.

A final point to consider in this comparison is that the rotational velocity (rpm) of the GFRE design is 3.68 times higher than the steel design. Often times one will hear that composite flywheels using magnetic bearings are more reliable than lower speed flywheels employing mechanical (rolling element) bearings. The fact is that because of the higher speeds required for maximizing the capability of composite materials, magnetic bearings are as much an enabling technology for using the GFRE materials as they are a factor in the system reliability. A comparison between the reliability attributes of rolling element bearings and magnetic bearings is the subject of Active Power white paper #111 (‘Quantitative Reliability Assessment of Ball Bearings versus Active Magnetic Bearings for Flywheel Energy Storage Systems).

* Energy density is also often expressed in terms of energy per unit volume in which case this would be the volumetric energy density. Where the preceding adjective is not provided, the intended context may be determined from the denominator of the units (i.e. Energy/mass or energy/volume, etc).

Two-Dimensional Flywheel Design

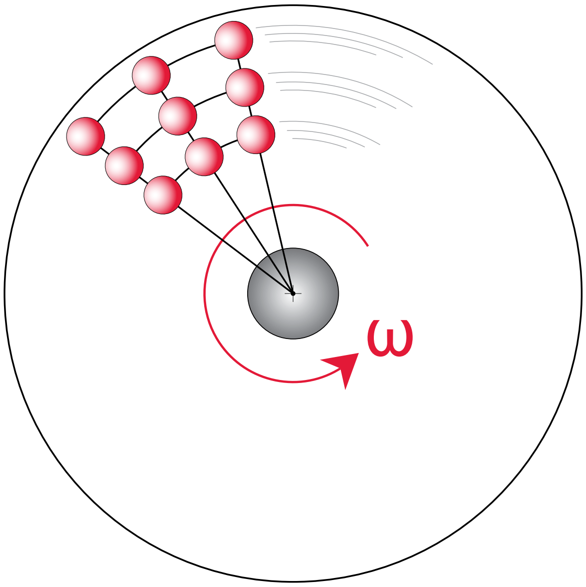

Figure 2: Two-dimensional flywheel extension from the one-dimensional representation.

Once the relationship between the energy storage and strength limitations of flywheel materials in one dimension has been visualized, it is a simple matter to extend this vision to two dimensional flywheels as shown in Figure 2. Instead of strings attaching the masses, stresses are applied to faces of adjacent material elements as shown in Figure 3. In fact, the representation here of small elements with surface stresses is exactly how engineers analyze components like flywheels using finite element analysis as shown in Figure 4. In the same analogy as the string breaking if too much force is applied, the same breakage occurs in a monolithic part if the applied stress exceeds the material strength, both expressed in units of force per unit area (i.e., pounds per square inch or Newton's per square meter, etc.).

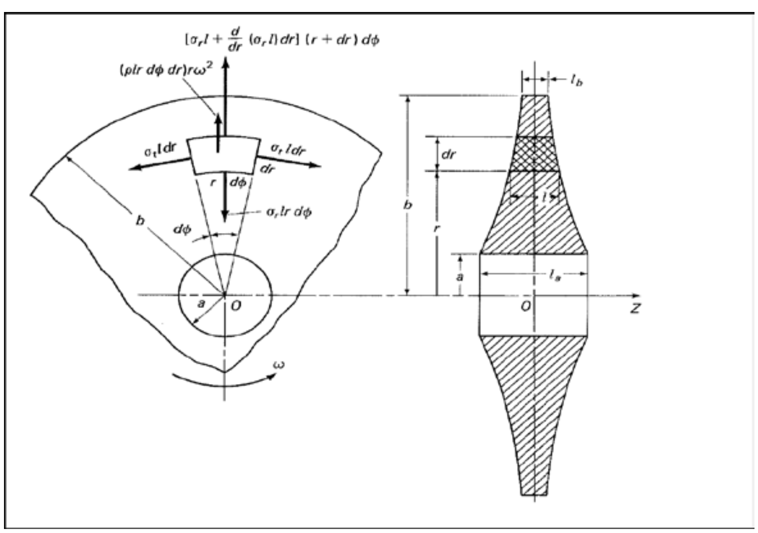

Figure 3: Two-dimensional stresses in a rotating disk. (Burr[1] ,1982).

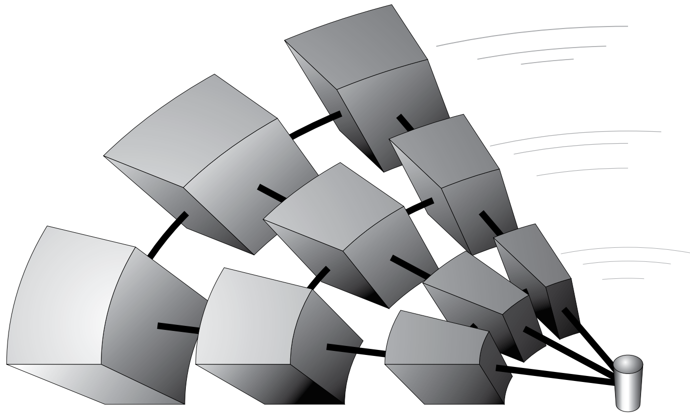

Figure 4: Finite element representation of a two-dimensional flywheel.

Elements radiating from the center outward are aligned in the *radial direction* and those aligned perpendicular to the radial elements are aligned in the *tangential* or *hoop* direction. In this case, the elements share centrifugal loads with neighboring elements in both the radial and tangential direction and the distribution of these loads depends to a great extent on the elasticity of the material. With steel flywheels both the elasticity and strength properties are relatively uniform in all directions. Composite materials can be designed with unique directional properties and designers take advantage of this when developing composite flywheels by aligning the fibers in the direction of highest loading. While some composite flywheel concepts have used fibers aligned in the radial direction, the predominant method is to align them in the hoop or tangential direction due to ease of manufacturing.

If one integrates the contribution of individual mass elements distributed around the axis of rotation of the flywheel, one obtains a property that has units of mass times radius squared as did the simple mass on a string which is given the name *polar moment of inertia* and is represented by the term J. For a straight, solid cylindrical flywheel having outer radius, r_o, the polar moment of inertia is:

$$J = \frac{1}{2} \cdot m \cdot r_o^2 \tag{5}$$

The mass of the straight, circular cylinder having density, ρ, can be computed as follows:

$$m = \pi \cdot \rho \cdot r_o^2 \tag{6}$$

Thus, the polar moment of inertia is:

$$J = \frac{1}{2} \cdot \pi \cdot \rho \cdot r_o^4 \tag{7}$$

The energy storage for the same geometry flywheel is:

$$E_{stored} = \frac{1}{2} \cdot J \cdot \omega^2_{max} \tag{8}$$

The design of the flywheel motor-generator and/or control electronics for delivering power to and from the flywheel usually limits the minimum speed of the flywheel so the useful energy is less than the stored energy. The speed ratio ‘k’ is defined as the ratio of the maximum speed to the minimum speed. In addition, the discharge efficiency, η, of the flywheel motor-generator and power converter further limits the energy delivered to the load.

The energy extracted from the flywheel by the motor-generator is given by:

$$E_{extracted} = \frac{1}{2} \cdot J \cdot (\omega^2_{max} \cdot \omega^2_{min}) = \frac{1}{2} \cdot J \cdot \omega^2_{max} \cdot \left[1 - \left(\frac{1}{k}\right)^2\right] \tag{9}$$

The energy delivered to the load is given by:

$$E_{load} = \eta \cdot \frac{1}{2} \cdot J \cdot \omega^2_{max} \cdot \left[1 - \left(\frac{1}{k}\right)^2\right] \tag{10}$$

Substituting equation 7 into equation 10 yields:

$$E_{load} = \eta \cdot \frac{1}{2} \cdot \pi \cdot \rho \cdot r_o^{\,2} \cdot (r_o \cdot \omega_{max})^2 \cdot \left[1 - \left(\frac{1}{k}\right)^2\right] \tag{11}$$

The first term in brackets is referred to as the “tip-speed”, peripheral speed or surface speed of the rotor and it has the units of velocity (inches/second or meters/second) as expected. The delivered energy capacity of the flywheel is proportional to the tip speed squared. The reason for presenting the flywheel energy relationship in this manner will become apparent when one evaluates the stresses in the same geometry.

The maximum tangential stress in a solid circular disk having material density, ρ, a material constant† υ, and spinning with angular velocity, ω, is given by:

$$\sigma_t = \rho \cdot \frac{3 + \nu}{8} \cdot (r_o \cdot \omega)^2 \tag{12}$$

Equation 12 shows the stress developed in the spinning flywheel is proportional to the material density and the tip-speed. Thus, one cannot arbitrarily increase the speed without compromising the safety of the flywheel. One final equation is worth considering. If Equation 11 is divided by Equation 6, the expression for the gravimetric energy density of the two-dimensional flywheel is realized:

$$\frac{E}{m} = \frac{1}{4} \bullet (r_o \bullet \omega_{max})^2 \bullet \left[1 - \left(\frac{1}{k}\right)^2\right] \quad (13)$$

Equation 13 can be rewritten in terms of tip-speed squared and substituted into Equation 13. The result will show the energy density (energy per unit mass) of the flywheel is proportional to the specific strength (strength per unit density) of the material being used.

$$\frac{E}{m} \propto \frac{\sigma_t}{\rho} \quad (14)$$

This relationship is presented as a fundamental attribute of flywheel energy-storage systems in Genta[2] (1985) and shows that, as with the one-dimensional flywheel, the primary advantage derived from using materials like GFRE composites with high specific strength is the improved gravimetric energy density. Furthermore, for materials such as steel it should be evident that it is not the rotational speed or angular velocity that determines the available energy; rather, it is the limiting tip-speed that is dependent on the allowable stress that defines the overall design and rotor weight.

As an example, the above relationships were built into an Excel spreadsheet and analyzed based on the power and runtime of one high-speed steel flywheel manufacturer who claims that high-speed operation results in a lower weight design. The results of this analysis are shown in Table 2 and indicate that regardless of rotor speed, the flywheel weight is the same. The accompanying chart shows that as the rotational speed of the rotor increases, the outer radius of the rotor must decrease to maintain constant tip speed. The only way then to obtain the desired energy is to increase the rotor length. However, the resulting flywheel mass is the same for all configurations. Interestingly, this manufacturer uses a flywheel with high length to diameter ratio and operates at high rotational speed. This high speed operation requires the use of magnetic bearings which in a separate white paper can be shown to have lower long term reliability than grease lubricated ball bearings when the ball bearings are properly designed.

Low speed steel flywheels supported on ball bearings have been performing admirably in aerospace gyroscopes for decades. The International Space Station (ISS) Control Moment Gyro (CMG) system uses 220 pound thin disk stainless steel flywheels rotating at 6,000-rpm on lubricated ball bearings. A picture of this system is shown in Figure 5. The U.S. Navy only recently (2005) posted a broad agency announcement (BAA) for research and development on advanced CMG systems with the primary goal of reducing system weight[3]. At an estimated launch cost of $10,000 per pound, composite materials make sense for this application since weight is such a critical element. However, for more earthly applications, whether to use composite materials and their associated weight reduction or steels and their long-term history of success boils down to more fundamental questions such as the final product cost, performance and reliability.

† The material constant, υ, is called Poisson's ratio and represents a coupling factor between deformations in one axis that influence deformations in another axis. For metals, the Poisson ratio is typically 0.29 and is relatively uniform in all directions. Think of a sheet of rubber that is pulled uniformly on two opposing edges. The rubber will elongate in the direction that the sheet is pulled and contract perpendicular to that direction. The ratio of the contraction to pull will be 29 percent if the rubber has the same Poisson ratio as steel. Composite materials can have dramatically different properties in each of the three principle material dimensions.

Power	140 kW
Speed ratio	2.00:1
Discharge efficiency	95%
Discharge time	15 sec
Delivered energy	2,100 kWs
Stored energy	2,947 kWs
UTS	180 ksi 1,241.1 MPa
UTS safety factor	3:1
Design stress	60.0 ksi 413.7 MPa
Poisson ratio	0.29
Poisson factor	0.41
Density	0.283 lb/in^3 7,833.5 kg/m^3
Tip speed	1,175.40 ft/s 358.35 m/s

radius		speed		length		l/d	mass	
(in)	(m)	rad/s	rpm	(in)	(m)		(lb)	(kg)
3	0.075	4,703	44,908	25.3	0.64	4.22	202	92
4	0.102	3,527	33,681	14.2	0.34	1.78	202	92
5	0.127	2,822	26,945	9.1	0.23	0.91	202	92
6	0.152	2,351	22,454	6.3	0.16	0.53	202	92
7	0.178	2,015	19,246	4.6	0.12	0.33	202	92
8	0.203	1,764	16,841	3.6	0.09	0.22	202	92
9	0.229	1,568	14,969	2.8	0.07	0.16	202	92
10	0.254	1,411	13,473	2.3	0.06	0.11	202	92
13	0.330	1,085	10,363	1.3	0.03	0.05	202	92
14	0.356	1,008	9,623	1.2	0.03	0.04	202	92
15	0.381	941	8,982	1.0	0.03	0.03	202	92
16	0.406	882	8,420	0.9	0.02	0.03	202	92
17	0.432	830	7,925	0.8	0.02	0.02	202	92
18	0.457	784	7,485	0.7	0.02	0.02	202	92
19	0.483	743	7,091	0.6	0.02	0.02	202	92
20	0.508	705	6,736	0.6	0.01	0.01	202	92

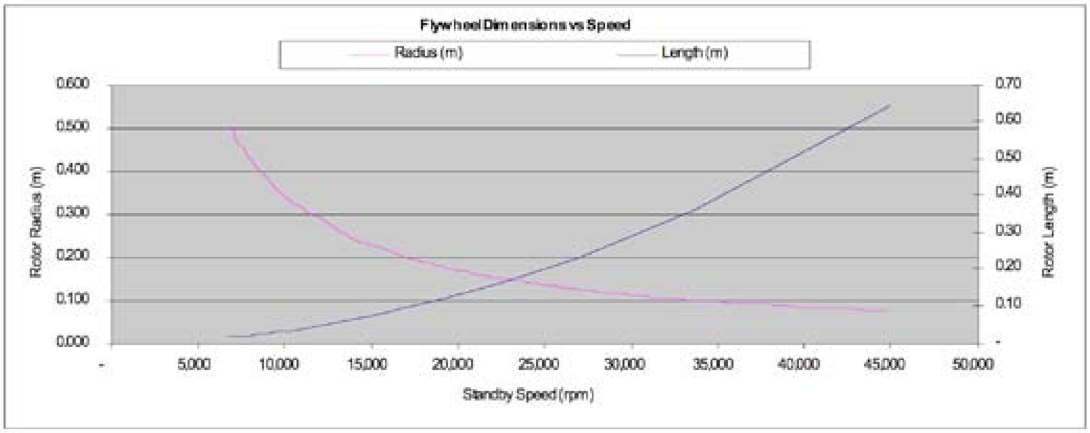

Table 2: Sizing of a straight cylindrical flywheel based on typical steel properties. Note that the flywheel weight is independent of speed.

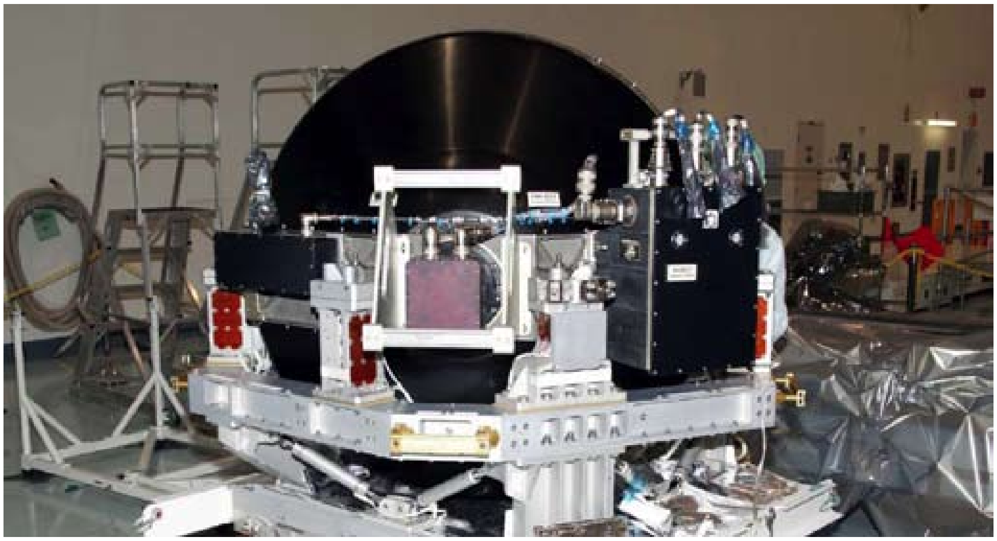

FIGURE 5: 220 POUND, 6,000 RPM STAINLESS STEEL CONTROL MOMENT GYROSCOPE (CMG) FLYWHEEL USED ON THE INTERNATIONAL SPACE STATION (ISS)[4].

SUMMARY

An understanding of the relationship between flywheel mass properties and strength properties can be obtained by looking at the simple concept of a massive ball attached to a string having a limited tensile strength. From this analogy, the predominant advantage of composite materials is the increased energy density (i.e., energy per unit weight.). The analysis is then extended to two-dimensional flywheel geometries that show a similar relationship between flywheel weight, the specific strength of the flywheel material, and the relationship to rotor surface speed, also known as tip speed.

Where weight is a design parameter that must receive high priority, composite materials offer potential for performance enhancement. This benefit comes at the expense of higher material costs. It is erroneous to assume that high speeds are a prerequisite for high performance flywheels when certain attributes such as cost, robustness of design and reliability may be of higher value.

When designing flywheels for UPS applications, the choice between composite materials and their associated weight reduction or steels and their long term history of success typically boils down to fundamental issues such as the final product cost, performance and reliability. In his book *Kinetic Energy Storage*, G. Genta summarizes: “A reliable, safe, well-designed and well-built medium energy-density rotor is enough for most applications.”

Appendix A – What is Angular Velocity?

Thinking back to high school geometry, one may recall that the ratio of the circumference to the diameter of a circle was known by ancient Egyptian and Greek geometers to be the same for all circles and that the value was slightly more than 3. The actual value has since been calculated more accurately and is 3.14159 to five significant digits. The Greek letter π (pronounce pi) is used to represent this value.

If one walks around a path that prescribes a complete circle, the distance one has traveled, *regardless of the size of the circle*, is π or 3.14159 units (these units being dimensionless). It is only when π is multiplied by the diameter (or 2π by the radius) of the circle that the actual (dimensional) distance traveled can be determined.

Engineers like to use names for units of measure like inches and feet. Just as a foot can be divided into smaller units of measure, circular units of measure can be divided into smaller increments. Whereas one twelfth of a foot is called an inch, $1/2\pi$ (1/6.28319) of a circular unit of measure is called a *radian*. Thus, there are 2π radians in a circle, π radians in a half-circle, and $\pi/2$ radians is a quarter circle; again independent of the size of the circle.

Velocity is equal to distance divided by time. Linear velocity is equal to, the linear distance traveled divided by the time that it takes to travel this distance. Angular velocity is equal to the angular distance traveled (measured in units of radians as discussed above), divided by time. Angular velocity will be expressed in units of *radians per second* as opposed to linear velocity which is expressed in units of *inches per second*.

If one now considers a flywheel rotating at 7,700 revolutions per minute, what is the angular velocity? Remembering there are 2π radians in the circumference of a circle and one revolution defines a complete circular path, it can be concluded that there 2π radians per revolution times 7,700 revolutions equaling 48,381 radians traveled by this flywheel in one minute of operation. Given 60 seconds in a minute, the angular velocity for this flywheel can also be expressed as 806.3 radians per second. Note again that at this time the size of the flywheel has not been mentioned.

Why is this important for flywheels? The primary reason is that the flywheel mass is distributed over its volume so not all of the mass is moving at the same speed. If one considers the flywheel as being divided into small, interconnected and equal sized lumps of matter, the lumps at the outer radius are moving faster and thus store more energy than the equal sized lumps at the inner radius. In order to determine the stored energy of the flywheel, the contribution of the individual elements needs to be calculated and then summed them together. Fortunately, engineers have some mathematical tools that allow this to be done analytically.

References

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